

Cosmic ray orbits in interplanetary magnetic fields

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With 8 figures in the text

ABSTRACT

An interplanetary magnetic field, which should agree with observed cosmic ray and magnetic storm effects, has been suggested by Alfvén. This field is worked out more in detail in the present paper, and several cosmic ray orbits are calculated in this field. The results are briefly discussed and compared with cosmic ray observations. A beam field of 30μ gauss at the earth's orbit has been assumed here, and according to Venkatesan's analysis of Forbush decreases the beam fields may vary from a few to about 100μ gauss.

Introduction

Some time ago Alfvén (1956) suggested a type of interplanetary magnetic field, which might be compatible with observed cosmic ray and magnetic storm effects. It should be of great interest to calculate cosmic ray orbits in this field so that more detailed comparisons with observations in nature can be made. The numerical integration has now been carried out at the Stockholm electronic computer BESK.

We introduce a rectangular coordinate system with the origin in the centre of the sun, the x - and y -axis in the equatorial plane, where the beam is centered around the y -axis, and the z -axis towards the magnetic south pole (approximately the heliographic north pole). Spherical coordinates will also be used. The coordinate system is shown in Fig. 1.

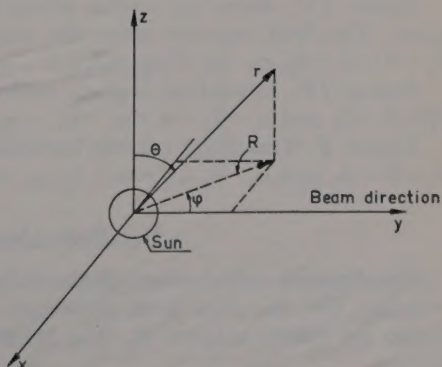


Fig. 1. The coordinate system.

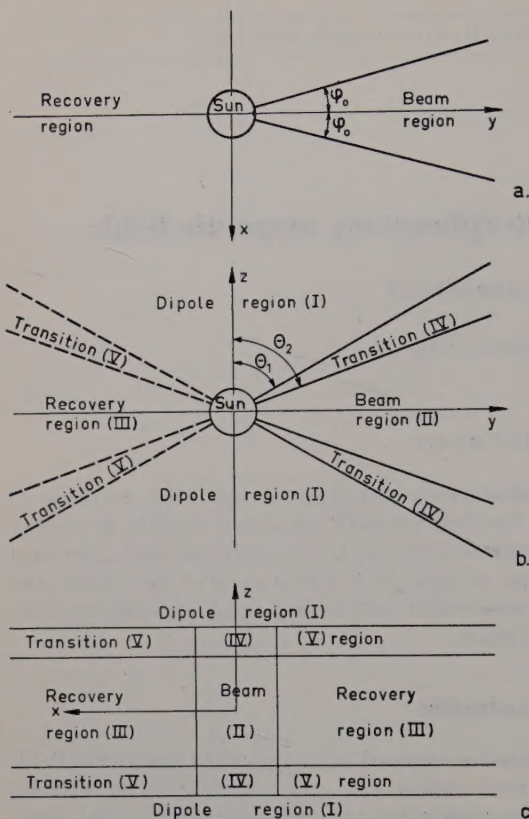


Fig. 2. The different regions of the interplanetary magnetic field. (a) View from above the northern hemisphere of the sun. (b) A meridional plane. (c) View inside the beam towards the sun.

The magnetic field can be divided into different regions as shown in Fig. 2. It is assumed that there are no toroidal (φ -) components of the field. The definitions of the regions are as follows.

Region I. Polar regions at $0 < \theta < \theta_1$ and $\pi - \theta_1 < \theta < \pi$. The field is a dipole field.

Region II. Beam region where an outward transport of flux takes place. B_θ is assumed to be proportional to r^{-1} . $\theta_2 < \theta < \pi - \theta_2$, $-\varphi_0 < \varphi < \varphi_0$.

Region III. Recovery region where an inward transport of magnetic flux takes place. The field recovers when the strongly perturbing beam has passed. $B_\theta \sim r^{-2}$. $\theta_2 < \theta < \pi - \theta_2$, $\varphi_0 < \varphi < 2\pi - \varphi_0$.

Region IV. Transition regions between the beam region and the polar regions. $\theta_1 < \theta < \theta_2$ and $\pi - \theta_2 < \theta < \pi - \theta_1$, $-\varphi_0 < \varphi < \varphi_0$.

Region V. Transition regions between the recovery region and the polar regions. $\theta_1 < \theta < \theta_2$ and $\pi - \theta_2 < \theta < \pi - \theta_1$, $\varphi_0 < \varphi < 2\pi - \varphi_0$.

Detailed calculations of the field

For detailed calculations of the field we may use the fact that $\text{div } \mathbf{B} = 0$. We consider the sun as a homogeneously magnetized sphere, and the magnetic field inside the sphere may be B_0 . The MKSA-unit system will be used. The sun's radius is $r_\odot$.

Regions I

Polar regions with dipole field. The components of the field are

$$\left. \begin{aligned} B_r &= -B_0 \cos \theta (r/r_\odot)^{-3}, \\ B_\theta &= -B_0 \frac{1}{2} \sin \theta (r/r_\odot)^{-3}, \\ B_\varphi &= 0. \end{aligned} \right\} \quad (1)$$

Region II

Beam region. B_θ is proportional to r^{-1} . Let us put

$$B_\theta = -K_1 r^{-1}. \quad (2)$$

Since B_φ vanishes according to our assumptions $\text{div } \mathbf{B} = 0$ gives

$$r^{-2} \frac{\partial}{\partial r} (r^2 B_r) + (r \sin \theta)^{-1} \frac{\partial}{\partial \theta} (B_\theta \sin \theta) = 0. \quad (3)$$

This gives

$$r^2 B_r = r_\odot^2 B_r^\odot - \int_{r_\odot}^r \frac{r}{\sin \theta} \frac{\partial}{\partial \theta} (B_\theta \sin \theta) dr. \quad (4)$$

$B_r^\odot = -B_0 \cos \theta$ is the radial component of the field at the sun's surface. Thus, in region II we get

$$\left. \begin{aligned} B_r &= -B_0 \left(\frac{r_\odot}{r} \right)^2 \cos \theta + K_1 \frac{r - r_\odot}{r^2} \cot \theta, \\ B_\theta &= -K_1 r^{-1}, \\ B_\varphi &= 0. \end{aligned} \right\} \quad (5)$$

Region III

Recovery region. Starting from the basic assumption that $B_\theta \sim r^{-2}$, we obtain in the same way as in region II

$$\left. \begin{aligned} B_r &= -B_0 \left(\frac{r_\odot}{r} \right)^2 \cos \theta + K_2 r^{-2} \cot \theta \log \frac{r}{r_\odot}, \\ B_\theta &= -K_2 r^{-2}, \\ B_\varphi &= 0. \end{aligned} \right\} \quad (6)$$

Regions IV and V

Transition regions. The field is according to our assumptions only determined by the following facts:

- (a) $B_\varphi = 0$.
- (b) $\text{div } \mathbf{B} = 0$.

Condition (b) makes it possible to connect the polar regions (I) with the equatorial regions (II and III) by means of field lines. For example, if we know that a certain field line enters the region at $r = r_2$, $\theta = \theta_2$ at the border of an equatorial region, we can find the point $r = r_1$, $\theta = \theta_1$ where this field line leaves the region. However, this is not enough to determine the field in detail. We need another assumption, and this may be made in such a way that it facilitates the calculations. We assume that the field lines are determined by

$$r = r_1 + (r_2 - r_1) \frac{\sin \theta - \sin \theta_1}{\sin \theta_2 - \sin \theta_1}. \quad (7)$$

Fig. 3 shows the field lines in regions IV and V qualitatively. Field lines through $r = r_\odot$, $\theta = \theta_1$ divide the transition regions into two different parts, *a* and *b*,

(a) where the flux leaves the region through the cone $\theta = \theta_1$ (on the northern hemisphere), and

(b) where the flux leaves the region through the solar surface.

The field lines through $r = r_\odot$, $\theta = \theta_1$ reach the cone $\theta = \theta_2$ at $r = r_0$, say, where r_0 is determined by the flux condition

$$\int_{r_\odot}^{r_0} B_\theta 2\pi r \sin \theta_2 dr = -B_0 \pi r_\odot^2 (\sin^2 \theta_2 - \sin^2 \theta_1) \\ = \text{flux through solar surface between } \theta_1 \text{ and } \theta_2. \quad (8)$$

We get two different values of r_0 , one in region IV, r_0^{IV} , and one in region V, r_0^{V} , corresponding to $B_\theta = -K_1 r^{-1}$ (as in region II) and $B_\theta = -K_2 r^{-2}$ (as in region III), respectively. The equation (8) gives

$$r_0^{\text{IV}} = r_\odot + \frac{B_0 r_\odot^2}{2 K_1} \frac{\sin^2 \theta_2 - \sin^2 \theta_1}{\sin \theta_2}, \quad (9a)$$

$$r_0^{\text{V}} = r_\odot \exp \left\{ \frac{B_0 r_\odot^2}{2 K_2} \frac{\sin^2 \theta_2 - \sin^2 \theta_1}{\sin \theta_2} \right\}. \quad (9b)$$

By calculating the flux through the cones $\theta = \theta_1$ between $r = r_\odot$ and $r = r_1$, and $\theta = \theta_2$ between $r = r_0$ and $r = r_2$ we can get a connection between r_1 and r_2 i.e. between the points where a field line leaves and enters the transition regions IVa and Va (Fig. 3). At $\theta = \theta_1$ we take B_θ from (1). Thus, the flux through $\theta = \theta_1$ is

$$\phi_1 = \int_{r_\odot}^{r_1} B_{\theta_1} 2\pi r \sin \theta_1 dr = -\pi B_0 r_\odot^3 \sin^2 \theta_1 \left(\frac{1}{r_\odot} - \frac{1}{r_1} \right). \quad (10)$$

At $\theta = \theta_2$ in the regions II (IV) and III (V) we get the flux ϕ_2

$$\phi_2^{\text{IV}} = \int_{r_0^{\text{IV}}}^{r_2^{\text{IV}}} -K_1 r^{-1} 2\pi r \sin \theta_2 dr = -2\pi K_1 (r_2^{\text{IV}} - r_0^{\text{IV}}) \sin \theta_2, \quad (11a)$$

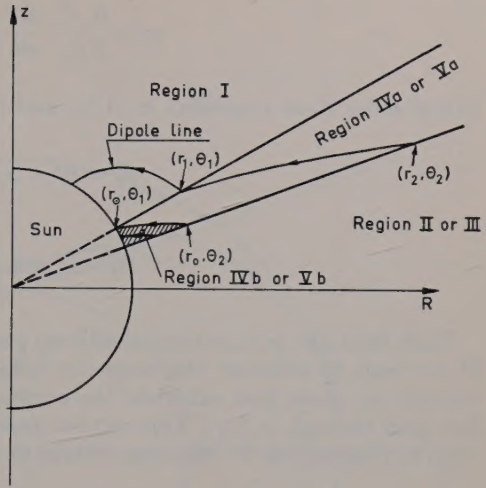


Fig. 3. Qualitative sketch of the magnetic field lines in the transition region.

$$\phi_2^V = \int_{r_0^V}^{r_2^V} -K_2 r^{-2} 2\pi r \sin \theta_2 dr = -2\pi K_2 \left(\log \frac{r_2^V}{r_0^V} \right) \sin \theta_2. \quad (11 b)$$

$$\phi_1 = \phi_2^{IV} \text{ gives}$$

$$r_2^{IV} = r_0^{IV} + \frac{B_0 r_\odot^2}{2K_1} \frac{\sin^2 \theta_1}{\sin \theta_2} \left(1 - \frac{r_\odot}{r_1^{IV}} \right) = r_\odot + \frac{B_0 r_\odot^2}{2K_1} \left\{ \sin \theta_2 - \frac{\sin^2 \theta_1}{\sin \theta_2} \frac{r_\odot}{r_1^{IV}} \right\}. \quad (12 a)$$

$$\phi_1 = \phi_2^V \text{ gives}$$

$$\begin{aligned} r_2^V &= r_0^V \exp \left\{ \frac{B_0 r_\odot^2}{2K_2} \frac{\sin^2 \theta_1}{\sin \theta_2} \left(1 - \frac{r_\odot}{r_1^V} \right) \right\} \\ &= r_\odot \exp \left\{ \frac{B_0 r_\odot^2}{2K_2} \left(\sin \theta_2 - \frac{\sin^2 \theta_1}{\sin \theta_2} \frac{r_\odot}{r_1^V} \right) \right\}. \end{aligned} \quad (12 b)$$

The values in MKSA units of the different constants may in a representative case according to Alfvén (1956) be

$$\begin{aligned} B_0 &= 1.2 \times 10^{-3} \text{ Vs/m}^2 = 12 \text{ gauss}, \\ K_1 &= 0.65 \times 10^{-6} r_\odot \text{ Vs/m}^2 r_\odot \text{ (30 } \mu \text{ gauss at the earth's orbit),} \\ K_2 &= 0.88 \times 10^{-4} r_\odot^2 \text{ Vs/m}^2 r_\odot^2 \text{ (19 } \mu \text{ gauss at the earth's orbit),} \\ \theta_1 &= 60^\circ, \\ \theta_2 &= 70^\circ, \\ \varphi_0 &= 15^\circ. \end{aligned}$$

We define some new constants:

$$\alpha_1 = \frac{B_0 r_\odot^2}{2K_1} \frac{\sin^2 \theta_1}{\sin \theta_2} = 736.7 r_\odot, \quad (13 a)$$

$$\alpha_2 = \frac{B_0 r_\odot^2}{2 K_2} \frac{\sin^2 \theta_1}{\sin \theta_2} = 5.442. \quad (13 \text{ b})$$

Introducing these constants in (12a) and (12b) we get

$$r_2^{\text{IV}} = r_0^{\text{IV}} + \alpha_1 \left(1 - \frac{r_\odot}{r_1^{\text{IV}}} \right), \quad (14 \text{ a})$$

$$r_2^{\text{V}} = r_0^{\text{V}} \exp \left\{ \alpha_2 \left(1 - \frac{r_\odot}{r_1^{\text{V}}} \right) \right\}, \quad (14 \text{ b})$$

Each field line in a certain meridional plane can be defined by the corresponding r_1 . If we want to calculate the magnetic field at the point (r, θ, φ) within the transition regions, we must first calculate the corresponding r_1 , i.e. we must know which field line goes through (r, θ, φ) . This can be made by combining (7) and (14a) for region IV or (14b) for region V. We then obtain in region IV

$$r_1^{\text{IV}} = \frac{\sin \theta_2 - \sin \theta_1}{2 (\sin \theta_2 - \sin \theta)} \left\{ -D + \sqrt{D^2 + 4 \alpha_1 r_\odot \frac{(\sin \theta_2 - \sin \theta) (\sin \theta - \sin \theta_1)}{(\sin \theta_2 - \sin \theta_1)^2}} \right\}, \quad (15)$$

$$\text{where} \quad D = -r + (r_0^{\text{IV}} + \alpha_1) \frac{\sin \theta - \sin \theta_1}{\sin \theta_2 - \sin \theta_1}, \quad (15 \text{ a})$$

and in region V, r_1^{V} is determined by

$$a - b x = e^{-1/x} \quad (16)$$

$$\text{with} \quad a = \frac{\sin \theta_2 - \sin \theta_1}{\sin \theta - \sin \theta_1} \frac{r}{r_0^{\text{V}}} e^{-\alpha_2}, \quad (16 \text{ a})$$

$$b = \frac{\sin \theta_2 - \sin \theta}{\sin \theta - \sin \theta_1} \frac{r_\odot}{r_0^{\text{V}}} \alpha_2 e^{-\alpha_2}, \quad (16 \text{ b})$$

$$x = \frac{r_1^{\text{V}}}{\alpha_2 r_\odot}. \quad (16 \text{ c})$$

These equations will be used also in the regions IVb and Vb (Fig. 3) where the field lines never reach the cone $\theta = \theta_1$. The values of r_1 obtained there have no physical meaning but they are necessary for the calculations.

After having computed r_1 we can calculate $\mathbf{B}$ in the following way. Consider a flux tube between the field lines r_1 and $r_1 + dr_1$, and between two arbitrary meridional planes separated by an angle of one radian. The flux in this tube through the cones $\theta = \theta_1$ and $\theta = \theta$ is determined by

$$B_\theta r \sin \theta dr = B_{\theta_1} r_1 \sin \theta_1 dr_1. \quad (17)$$

$$\text{Thus} \quad B_\theta = B_{\theta_1} \frac{\sin \theta_1}{\sin \theta} \frac{r_1}{r} \frac{dr_1}{dr}. \quad (18)$$

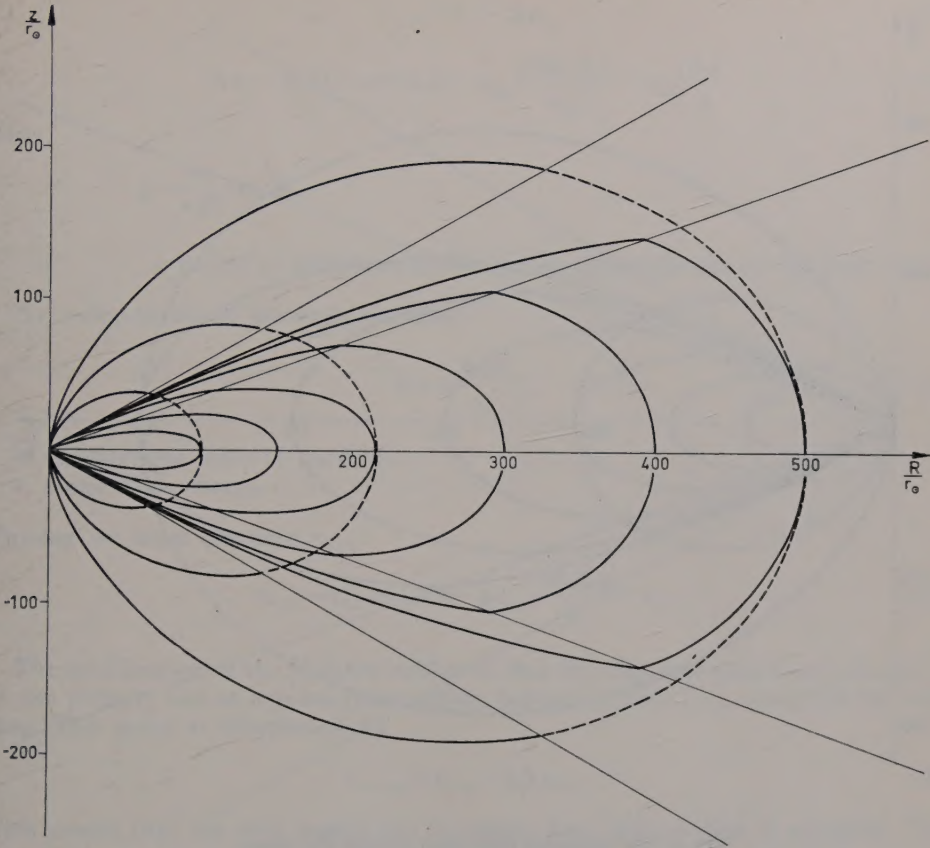


Fig. 4. The magnetic field lines in the beam.

B_{θ_1} is determined by equation (1) with $\theta = \theta_1$. In region IV we get from (7) and (14a)

$$\frac{dr}{dr_1^{\text{IV}}} = \frac{\sin \theta_2 - \sin \theta}{\sin \theta_2 - \sin \theta_1} + \frac{\alpha_1 r_0}{(r_1^{\text{IV}})^2} \frac{\sin \theta - \sin \theta_1}{\sin \theta_2 - \sin \theta_1}, \quad (19a)$$

In region V (7) and (14b) give

$$\frac{dr}{dr_1^{\text{V}}} = \frac{\sin \theta_2 - \sin \theta}{\sin \theta_2 - \sin \theta_1} + \frac{\alpha_2 r_0}{(r_1^{\text{V}})^2} \left(r - r_1^{\text{V}} \frac{\sin \theta_2 - \sin \theta}{\sin \theta_2 - \sin \theta_1} \right). \quad (19b)$$

B_r is determined by differentiating (7) since

$$B_r/B_{\theta} = dr/r d\theta \quad (20)$$

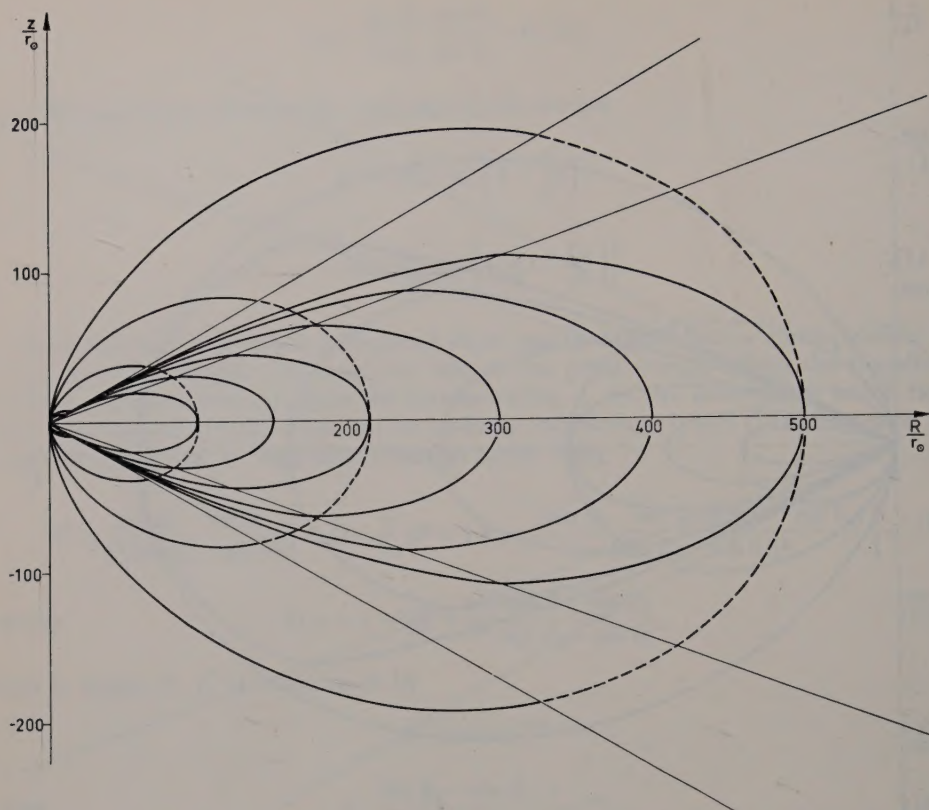


Fig. 5. The magnetic field lines outside the beam.

so that

$$B_r = B_\theta \frac{r_2 - r_1}{r} \frac{\cos \theta}{\sin \theta_2 - \sin \theta_1}, \quad (21)$$

and this applies both to regions IV and V.

The shape of the field lines is shown in Figs. 4 and 5.

Calculation of orbits

A modification of Störmer's method has been used for the orbit calculations.

The velocity vector $\mathbf{v}$ can be divided in two parts

$$\mathbf{v} = \mathbf{v}_\perp + \mathbf{v}_\parallel, \quad (22)$$

where $\mathbf{v}_\perp$ is the component perpendicular to the magnetic field, and $\mathbf{v}_\parallel$ is the component parallel to the field.

When the integration has been carried up to the point $\mathbf{r}_n = (x_n, y_n, z_n)$, where the velocity is $\mathbf{v}_n$, then the next point $\mathbf{r}_{n+1}$ is determined by

$$\mathbf{r}_{n+1} = \mathbf{r}_n + \Delta \mathbf{r}_n, \quad (23)$$

$$\Delta \mathbf{r}_n = \rho_n (1 - \cos \Delta \alpha) + \mathbf{v}_{n\perp} \frac{\rho \sin \Delta \alpha}{v_{n\perp}} + \mathbf{v}_{n\parallel} \frac{\Delta s}{v}, \quad (24)$$

with

$$\rho = \frac{m}{e B^2} (\mathbf{v} \times \mathbf{B})$$

$$= \text{radius of curvature of the motion perpendicular to the field.} \quad (25)$$

Δs = step length of arc along the orbit.

$$\Delta \alpha = \frac{\Delta s v_{n\perp}}{\rho v}, \quad (26)$$

m = relativistic mass of particle,
 e = electronic charge.

Further we must compute $\mathbf{v}_{n+1}$:

$$\mathbf{v}_{n+1} = \mathbf{v}_{n\perp} \cos \Delta \alpha + \rho v_{n\perp} \frac{\sin \Delta \alpha}{\rho} + \mathbf{v}_{n\parallel}. \quad (27)$$

The modification of the Störmer method is that the magnetic field is not calculated at the point $\mathbf{r}_n$ but at a point intermediate between $\mathbf{r}_n$ and $\mathbf{r}_{n+1}$, except at the first step. This point is determined by

$$\mathbf{r}_{n+1/2} = \mathbf{r}_{n-1} + \frac{3}{2} \Delta \mathbf{r}_{n-1}. \quad (28)$$

This means that the step length can be rather long without loss of accuracy. The step length Δs must be chosen by considering both the gradient of the field and the angle $\Delta \alpha$, which should not be too large. In the present orbit integrations

$$\Delta s = r/32, \quad (29)$$

where r = distance from the sun. This means that B does not change by more than 10 % in one step. If (29) would give $\Delta \alpha > 0.25$ radians, then Δs is determined by (26) with $\Delta \alpha = 0.25$.

Selection of orbits

The orbits calculated are proton orbits with energies 10^9 , 10^{10} and 10^{11} eV. Each orbit is calculated backwards, starting on the earth (neglecting the earth's field). The electric field in the beam has not been considered, because the effects of it are already known well enough, as will be explained later. The earth was situated at four different positions with respect to the beam (all of them in the equatorial plane) one in the centre of the beam (position *A*), one outside (opposite) the beam (position *B*) and two at the border of the beam (positions *C* and *D*). This is shown in Fig. 6. Since the beam rotates together with the sun in an anti-clockwise direction as seen in Fig. 6, position *D* corresponds to the time just before the beam overtakes the earth.

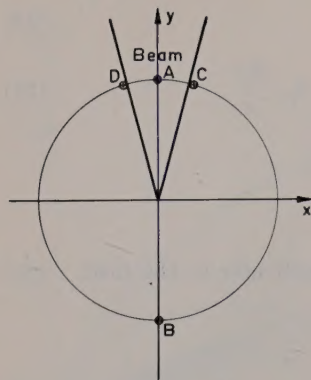


Fig. 6. The four positions of the earth relative to the beam.

Therefore, this is referred to as “before” the beam. For the same reason position *C* is referred to as “after” the beam. The direction of impact on the earth was, still neglecting the geomagnetic field:

$$\begin{aligned} \mathbf{v}_1 &= (0, 0, -v), \\ \mathbf{v}_{2,4} &= (\pm v/\sqrt{2}, 0, -v/\sqrt{2}), \\ \mathbf{v}_{3,5} &= (0, \pm v/\sqrt{2}, -v/\sqrt{2}), \\ \mathbf{v}_{6,10} &= (\pm v, 0, 0), \\ \mathbf{v}_{8,12} &= (0, \pm v, 0), \\ \mathbf{v}_{7,9,11,13} &= (\pm v/\sqrt{2}, \pm v/\sqrt{2}, 0). \end{aligned}$$

Thus, 3 energies, 4 positions of the earth and 13 directions of impact on the earth give 156 orbits. Each orbit has an identification number and the explanation of these numbers is given in Table 1.

In each group of 13 orbits (same energy and same position of the earth) the last 8 are confined in the equatorial plane. The first one is perpendicular to this plane when impinging on the earth, and the 4 others make an angle of 45° with it. The directions

Table 1. Explanation of the orbit numbers.

Particle energy eV	Position of the earth (see Fig. 6)			
	A (in the beam)	B (outside the beam)	C (after the beam)	D (before the beam)
10 ⁹	1-13	14-26	27-39	40-52
10 ¹⁰	53-65	66-78	79-91	92-104
10 ¹¹	105-117	118-130	131-143	144-156

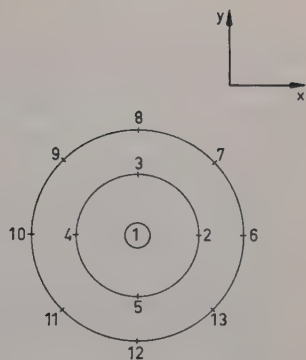


Fig. 7. The identification numbers of the orbits refer to the directions of impact on the earth. No. 1 comes perpendicular to the equatorial plane, 2-5 are inclined 45° and 6-13 are in the equatorial plane. To each number should be added 13 n , where $n = 0, 1, 2, \dots, 11$.

of the orbits with respect to the x - and y -axis (the beam is ejected in the y -direction) are indicated in Fig. 7.

The probable errors in the calculations are discussed in the Appendix.

Results

The results are given in the Plates I-XLIII. Some of the orbits are not given there, since they do not contribute anything essential to our knowledge.

The unit of length, L , in the plates is such that

$$L = 10.24 r_\odot.$$

The reason for this is that $1024 = 2^{10}$ is convenient to use in BESK. Hence, 1 A.U. = $215 r_\odot = 21.0 L$.

It is seen that the motion of the particles in trapped orbits is made up essentially of oscillations along the magnetic field lines from the northern to the southern hemisphere and back again, plus a drift around the sun, clockwise if seen from above the northern hemisphere. There is also a radial drift at the edges of the beam because of the different magnetic fields inside and outside the beam. It should be noticed, that for $r > 136 r_\odot = 13.3 L$, B is smaller outside than inside the beam. For $r < 136 r_\odot$ the opposite is true. Therefore, it may happen that a particle which goes almost perpendicular through the equatorial plane is displaced from the sun when leaving the beam, as is the case for orbit 27. There the main radial drift is caused by a displacement at $r < 136 r_\odot$ above the equatorial plane, where the field inside the beam is smaller than outside.

Discussion

From the results of the orbit calculations some conclusions may be drawn, which can be directly related to observations in nature.

At first some general remarks should be made. The mathematical model which has been treated here is an idealisation and in nature the following deviations from this model affect the results.

(a) There are electric fields in nature, in particular in the beam. The effects of this are, (i) a deceleration corresponding to the potential difference across the beam,

and (ii) a radial drift outwards from the sun with the velocity $v_b = E/B$ (beam velocity). The deceleration of the particles produces Forbush decreases when the earth is in such a position that it can be hit by decelerated particles. The radial displacement of a particle passing the beam due to the electric field is equal to the product of v_b and the time that it takes to pass the beam. This time is longer, the lower the energy is. In fact, it is inversely proportional to the kinetic energy perpendicular to the field, as long as the radius of curvature is small compared to the width of the beam ($W_\perp < 10^{10}$ eV, say). For higher energies the displacement is negligible. For particles with $W_\perp = 10^9$ eV at 1 A.U. from the sun the displacement is about 0.4 A.U. if $v_b = 2 \times 10^6$ m/s and if our assumptions about the magnetic field and the width of the beam are substantially correct.

The effect (i) of the electric field (the deceleration of cosmic rays) is of decisive importance for intensity variations of cosmic rays. Hence, an important part of the application of the present orbits to observations in nature is to investigate whether cosmic rays impinging on the earth from a certain direction at a certain time have recently traversed electric field regions (mainly beam regions) or not.

(b) The strength of the magnetic field may differ from that assumed here. In fact, Venkatesan (1957) has obtained values of the magnetic fields in the beams from studies of cosmic ray variations during magnetic storms. He has obtained values both from statistical means of many disturbances and from individual decreases. In the statistical means the intensity changes have been smoothed out and, hence, these result in much lower fields than the individual decreases. It seems reasonable to assume that the fields obtained from the individual decreases are more directly related to beam fields and, therefore, we will compare these with the assumed fields in the present model. For this we have to use Tables 6 and 7 in Venkatesan's paper. These tables give a very wide range of values, varying from time to time. The magnetic fields obtained are inversely proportional to the assumed velocity of the beam. Venkatesan has given values for $v_b = 2 \times 10^6$, 10^6 , 5×10^5 and 10^5 m/s. For 2×10^6 m/s the fields obtained range from 1.45×10^{-6} up to 78.4×10^{-6} gauss as compared to our value of 30×10^{-6} gauss. Now, 2×10^6 m/s is probably about the highest beam velocity, that ever occurs, and it is only reached at very strong storms. Hence, the lower field values, which refer to the weakest storms, should probably be connected with lower velocities, so that in reality the fields are not quite as low in these cases. It should also be noticed that Venkatesan has only obtained the magnetic field component perpendicular to the motion of the beam. The field may have other components, so the total field is larger. Nevertheless, it seems as if the beam fields assumed by Alfvén are rather high, so that they should be referred to rather heavy storms.

In view of this we may say that the magnetic fields may differ from those assumed here by a factor 3, up or down, say, or in extreme cases by a factor 10, at least down. If the field is a third of what has been assumed, the orbits correspond to energies of roughly 1.6×10^8 , 2.8×10^9 and 3.3×10^{10} eV, and if it is 3 times larger the energies are about 4.5×10^8 , 3×10^{10} and 3×10^{11} eV.

(c) The beams may not be symmetric around the equatorial plane, and they are certainly not as regular, and the borders are not as well defined as assumed here. In particular the surface currents connected with the sharp bends of the field lines at the border of the transition regions do not exist in reality. However, the latter is likely to be less important for the orbits than the electric field and the strength of the magnetic field.

(d) The earth may not be situated in the equatorial plane but some distance from it, viz. up to 7° , which is the inclination of the ecliptic to the sun's equator. The magnetic equator may also differ somewhat from the rotational equator. The beam may be ejected in such a direction that it does not hit the earth but passes over or under it. In such a case a Forbush decrease may occur without an associated magnetic storm.

(e) Here it has been assumed that the beam is straight in the y -direction whereas in reality it is curved because of the sun's rotation. It may very well be, that the field lines are curved in the same way too. We may say, that the magnetic meridional planes are curved. The direction of the curvature may change too, because if matter is moving inwards outside the beam this should result in a curvature in the opposite direction.

(f) The exponent of the field has been assumed to be constant ($=2$) in the whole recovery region. It is likely that it varies, so that it is larger, the longer the time since a beam passed over the region in consideration. This means e.g. that the exponent should be larger in the recovery region near position D (Fig. 6) than near position C .

(g) There may be no beam or several beams at the same time.

These differences should be kept in mind when applying the results of the calculations to observations in nature.

1. *The flare effect*

A remarkable flare effect occurred on Febr. 23, 1956. This has been studied among others by Meyer, Parker and Simpson (1956) and by Eckhartt (1957, 1958). The former assume a field free region between sun and earth so that the rays can easily move from the sun to the earth in the short time observed between the flare and the onset of the cosmic ray intensity increase. However, Eckhartt found that the direction of impact of the rays on the earth was not from the sun, but from a beam which produced a magnetic storm two days later. The beam appeared to emit cosmic rays (see Fig. 8). It is difficult to explain this if the region between sun and earth is field free. However, it is also difficult to explain it in terms of the present field, since it implies an anti-clockwise drift of the rays around the sun. Contrary to this all orbits integrated so far on the whole obey a clockwise drift direction. It is true, that for many orbits the clockwise drift is frequently interrupted by anti-clockwise drifts (see e.g. orbits 82 and 96, Pl. XXX and XXXI), but these are always so small that they cannot explain the effect observed by Eckhartt. Further, the anti-clockwise drift always takes place further from the equatorial plane than the earth is ever likely to be. This means that one would have to assume that the particles first drift anti-clockwise to a point where they can move along a field line towards the earth. However, then the rays impinging on the earth would appear to come from the sun. Hence, in this way the drift from the beam would be explained but not the observed direction of impact on the earth.

Eckhartt (1957) has worked out an explanation in terms of scattering by the beam of cosmic rays from the sun. He shows how 15 GeV particles can move from the beam to the earth if the field outside the beam at the earth's orbit is 1.9×10^{-6} gauss. This is a rather weak field, only one tenth of the field assumed in the present paper (although not inconceivably weak, in view of Venkatesan's result). The rays also

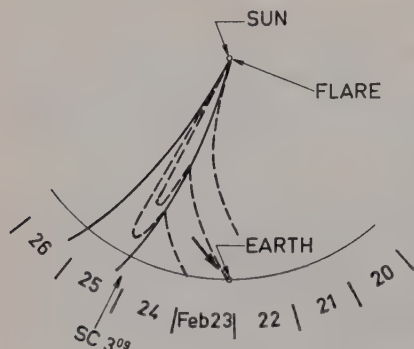


Fig. 8. The earth and the solar beam which is supposed to be responsible for the magnetic storm on Feb. 25, 1956. The direction of the cosmic rays impinging on the earth on Febr. 23 is shown by an arrow. This direction can be explained if the magnetic field lines are curved as shown by the dashed lines.

contain particles with lower energies, so actually one has to go down even more with the field. On the other hand, the field in the beam must be very strong in order to be able to scatter 15 GeV particles, at least as strong as assumed in this paper. This means a very strong gradient at the edge of the beam.

There is an alternative possibility, viz. that the field lines are not confined in meridional planes but that they are curved, about as shown by the dashed line in Fig. 8. Then we do not have to assume very weak magnetic fields, because even very low energy particles can drift along the field lines, and the correct impact direction on the earth is automatically obtained for particles of all possible energies.

2. Forbush decreases

It has been pointed out long ago (Brunberg and Dattner, 1954) that most of the cosmic ray particles, which reach the earth just after a beam has passed by, must have traversed the beam where they were decelerated by the electric field as explained earlier. This would cause Forbush decreases. In agreement with this it has also been shown by Venkatesan (1957) that during high solar activity there is a strong correlation between K_p and the negative time derivative of the cosmic ray intensity. Now it can be seen from the orbits 27–39 (Pl. X–XIV and XIX) and 79–91 (Pl. XXVII–XXXI and XXXVI) that in general trapped orbits have to pass the beam. However, at 10^{11} eV the orbits are not trapped and then not all of them have to pass the beam in order to reach the earth just after the beam has passed by. This can be seen from orbits No. 131–143 (Pl. XXXVII, XXXIX, XLII). The only orbits which definitely have to pass the beam are those reaching the earth from between the 1-hour and the 10-hour directions approximately. It is however, interesting to note, that all particles impinging from nearly the zenith direction at the magnetic poles of the earth must first traverse the beam. This can be seen from orbit 131, Pl. XXXVII, and it may imply that the Forbush decreases are more pronounced for higher energies at the poles than at lower latitudes, as seems to be the case according to measurements by Singer (1957) at Thule.

The fact that the cutoff energy in the geomagnetic field is lower at the poles than at lower latitudes should also result in heavier Forbush decreases, since the lower energy particles suffer more intensity decreases because of the larger percentage of losses in the beam. This consequence of the electric field theory has also been pointed out by Singer (1957). In that connection he states, however, that a latitude depend-

ance of the Forbush decreases has not been observed. This seems to be in contradiction to his statement that the Forbush decreases are twice as large at the poles as in lower latitudes. If he means that the decreases do not depend on the latitude except near the poles, he may be right, but it seems as if sufficient data about this are not yet available.

It is interesting to notice in Pl. XXXVII that even 10^{11} eV cosmic rays impinging in the zenith direction at the poles never go very far from the solar equatorial plane. The maximum distance from the equatorial plane for these orbits is not more than 0.5 A.U. except very far away (>2 A.U. from the sun) and the maximum solar latitude is about 25° . The same applies to many 10^{11} eV orbits inclined 45° to the equatorial plane (orbits 106–109, 119, 122, 133, 134, 146–148).

3. Diurnal variation during Forbush decreases

It has been remarked by Dattner and Venkatesan (1958), that the above mentioned fact, that a few orbits do not have to pass the beam in order to reach the earth after a storm, should cause a diurnal variation during Forbush decreases with a minimum around 6 h. They based their statement on orbits in the equatorial plane only. (Cf. orbits 136–143, Pl. XLII). However, also the orbits No. 132–135, Pl. XXXIX (inclined 45° with the equatorial plane) confirm their conclusion.

Diurnal variations in general have previously been discussed by Brunberg and Dattner (1954, Sandström (1956*a*, *b*), and by Dattner and Venkatesan (1958). The present model cannot directly account for most of these effects, since they depend on dynamic properties of the field and the cosmic ray gas, e.g. changes in the field with associated induced electric fields, diffusion due to density gradients and particle winds due to co-rotation with the sun. These effects may more successfully be studied by macroscopic magneto-hydrodynamic methods rather than by individual particle orbits.

Conclusion

The very brief discussion above indicates that the present model of the magnetic field may be a promising approach, which should be further investigated.

APPENDIX

Errors in the orbit calculations

The errors in the orbit calculations are of two types:

- (i) Errors due to the limited capacity of BESK's memory.
- (ii) Errors due to the numerical method of integration. These errors depend on the step length.

The errors of type (i) are negligible in comparison to the other errors. The program for the machine is made such that the relative error in any number can never be larger than 10^{-7} . By the relative error of a vector quantity we here mean the absolute error of this quantity divided by the modulus of the total vector.

Thus we can focus our interest on the error due to the method of integration.

Referring to eq. (23), let us for a moment assume that $\Delta \mathbf{r}_n$ is parallel to the x_i -axis. Then the error $d(\Delta x_j)$ is of interest only if $j = i$. If there were errors only along the orbit, a plotted calculated orbit would never deviate from the true orbit. It is only the directional error β in Δr_n which is important. Then

$$\beta = \frac{1}{\Delta x_i} \sqrt{\sum_{j \neq i} \{d(\Delta x_j)\}^2}. \quad (30)$$

There will also be a directional error γ in the new velocity vector $\mathbf{v}_{n+1}$ as calculated by (27). Omitting the index $n + 1$ we find

$$\gamma = v^{-1} \sqrt{\sum_{j \neq i} \{d v_j\}^2}, \quad (31)$$

so the total directional error per step will be

$$\delta = \sqrt{\beta^2 + \gamma^2}. \quad (32)$$

After n steps this will have increased to $\sqrt{n} \delta$. Thus, the total mean transverse error at the end of an orbit will be

$$\Delta = \sqrt{\sum_{n=1}^N \{\sqrt{n} \delta \Delta s\}^2} \approx N \delta \Delta s = \delta S, \quad (33)$$

where $S = N \Delta s$ is the total length of the orbit.

There are different cases with which we have to deal when determining δ . They depend on the five different regions of the field with which we are concerned and also whether the step length is determined by (29) or by $\Delta \alpha = 0.25$.

The calculations of δ are lengthy but quite straight forward from (24)–(27). In the two most important field regions the following approximate results can be given.

Beam region:

$$\begin{aligned} \delta_{11} &\approx 10^{-4} (W + 1) & \text{if } \Delta \alpha = 0.25 \\ \delta_{12} &\approx 10^{-2} / (W + 1) & \text{if } \Delta s = r/32 \end{aligned}$$

Recovery region:

$$\begin{aligned} \delta_{21} &\approx 10^{-6} \frac{r}{r_\odot} (W + 1) & \text{if } \Delta \alpha = 0.25 \\ \delta_{22} &\approx \frac{r_\odot}{r} \frac{5}{W + 1} & \text{if } \Delta s = r/32 \end{aligned}$$

Here W is the energy in GeV. $W + 1$ is a factor which is proportional to the mass of the protons, which happens to have a rest mass close to 1 GeV. At 1 A.U. from the sun the limit, where $\Delta \alpha = 0.25$ gives the same step length as $\Delta s = r/32$, is at about $W = 10$ GeV. Hence, as an average $\Delta \alpha = 0.25$ is used for $W < 10$ GeV and $\Delta s = r/32$ for $W > 10$ GeV. This means that the maximum errors occur for about

10 GeV. Thus for $r = 200 r_{\odot} \approx 1$ A.U. and for 10 GeV δ is of the order of 10^{-3} or 2×10^{-3} . Since the orbits are about $1000 r_{\odot}$ long, the errors will be one or a few solar radii according to (33).

In the case of orbits entirely in the equatorial plane the true orbits are well known, so a comparison with the computed orbits is easy to make. The results are typed by BESK with 4 digits, where one unit in the last digit corresponds to $0.1 r_{\odot}$. Thus, the smallest deviation from the true orbits, which can be detected, is $0.1 r_{\odot}$. Of 32 orbits calculated in the equatorial plane at 10^9 eV only one deviated as much as this, and in this case the deviation was $0.5 r_{\odot}$ towards the sun and no deviation in the z -direction. This was for orbit No. 6. Here the applicable angular deviation is $\delta_{11} \approx 2 \times 10^{-4}$ so that the mean deviation after $10^3 r_{\odot}$ is of the order of $0.2 r_{\odot}$.

The largest errors should occur for 10^{10} eV. Of the 32 orbits in the equatorial plane for this energy two showed no detectable deviation, namely No. 84 and 91. The mean error for orbits at 10^{10} eV should be of the order 1 – $2.5 r_{\odot}$. Thus, the orbits 84 and 91 deviate by less than one tenth of the probable deviation. No orbits show any detectable deviation when half of the integration is performed. After this they deviate more, sometimes even more than $10 r_{\odot}$ but on the whole our estimate of the errors seems to be confirmed.

For 10^{11} eV no deviations have been detected. Thus, it is seen that for 10^{10} eV the errors are so large that $10^3 r_{\odot}$ is about the longest orbit that can be safely integrated with this method.

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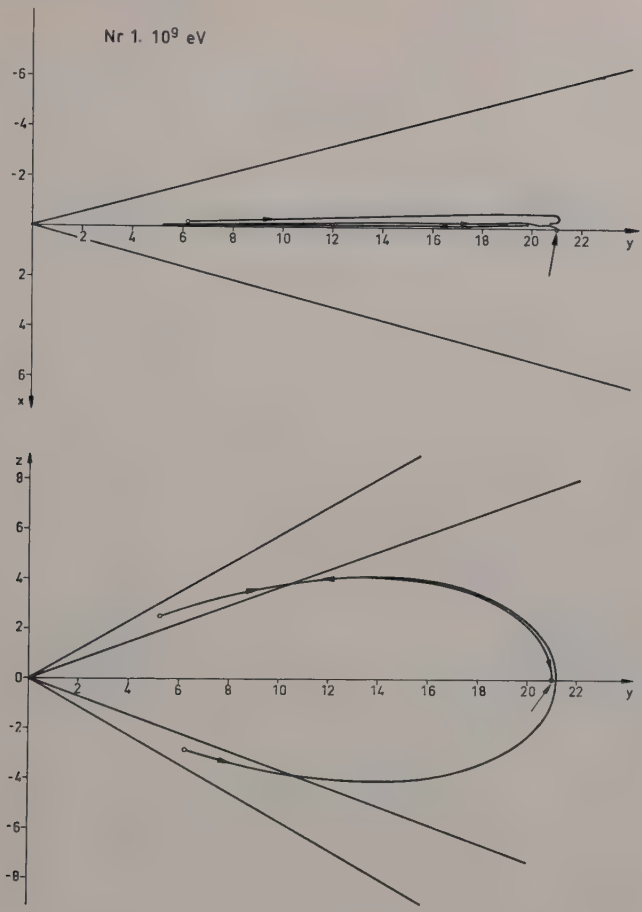
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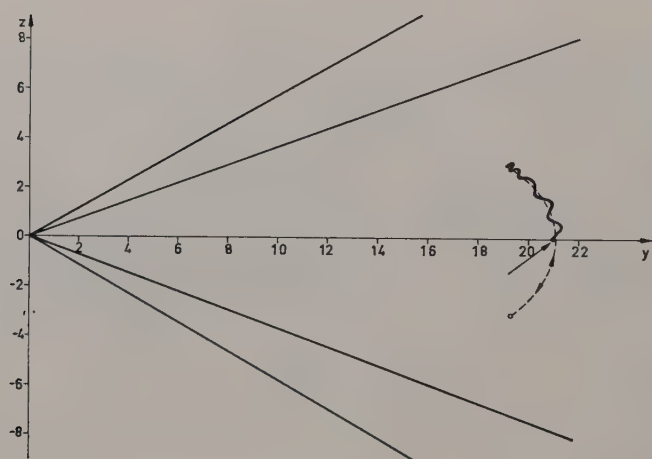
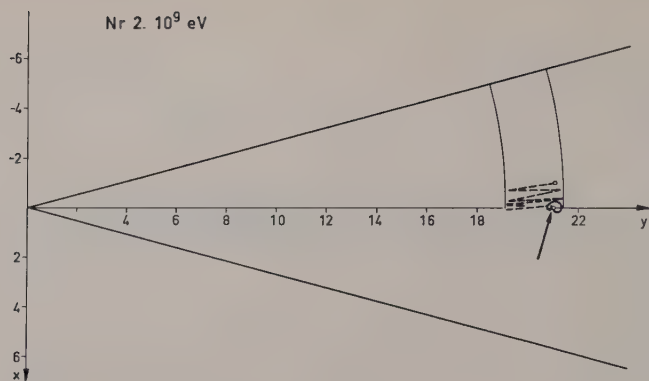
Plates I–XLIII

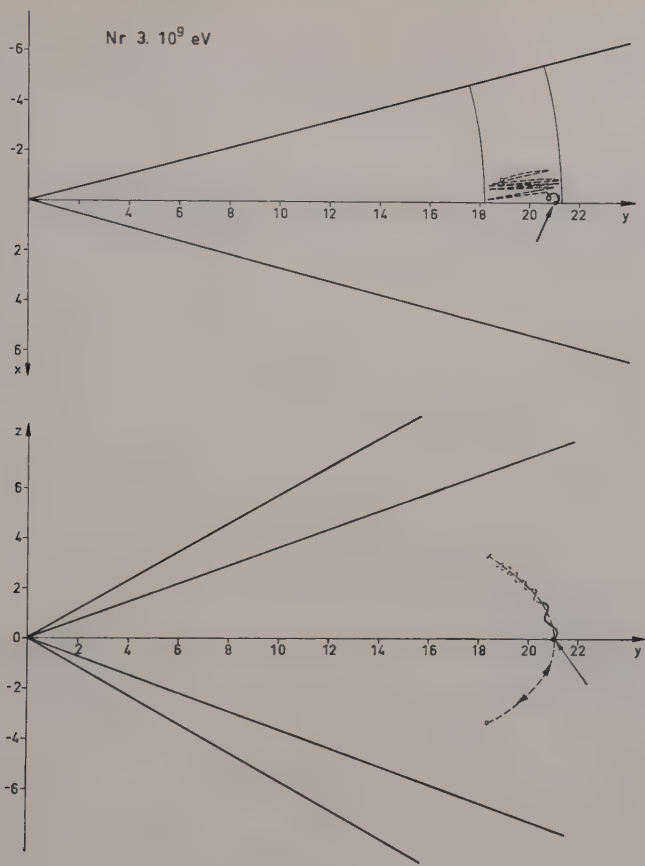
The position of the earth is indicated by an arrow, except in a few cases where this would only confuse. The start and end points of each orbit are indicated by rings. Except in Plates XLII and XLIII (see below) dashed lines indicate approximate or simplified orbits, e.g. drift orbits. Examples of this are 10^9 eV orbits which oscillate along the magnetic field lines at the same time as they drift in the $-\varphi$ -direction. In this case the dashed lines indicate the approximate amplitude of the oscillations.

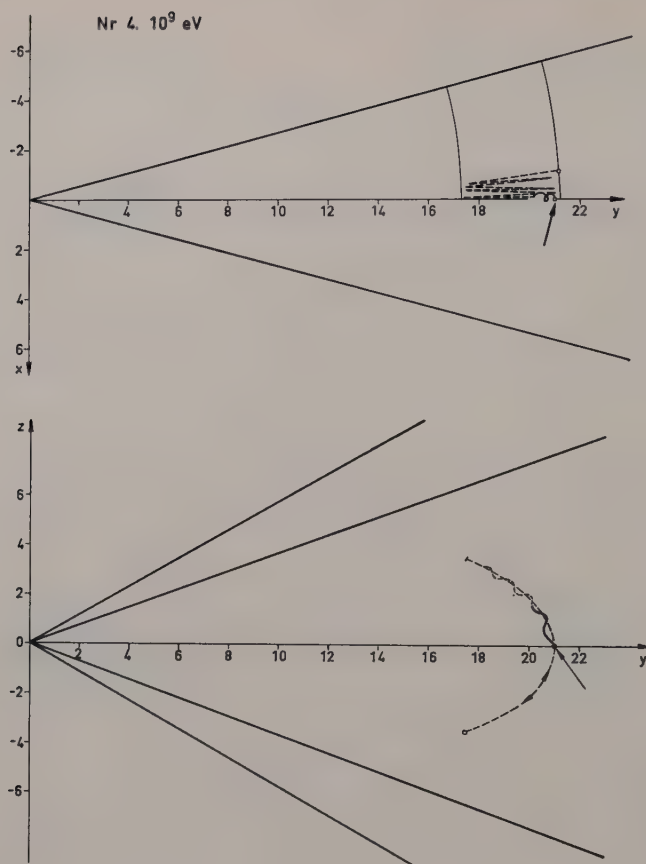
Many 10^{10} eV orbits are so complicated that it would be impossible to trace the orbits if they were plotted in their whole length. In these cases a part in the middle has been omitted. This omitted part may contain a few oscillations along the magnetic field lines. For orbit 19 only one point (indicated by a ring) on each loop of the trochoid has been given. The distance between two consecutive rings indicates the drift per revolution.

The dashed lines in Plates XLII and XLIII indicate orbits not directly integrated but copied from corresponding orbits with the earth in position *B*.

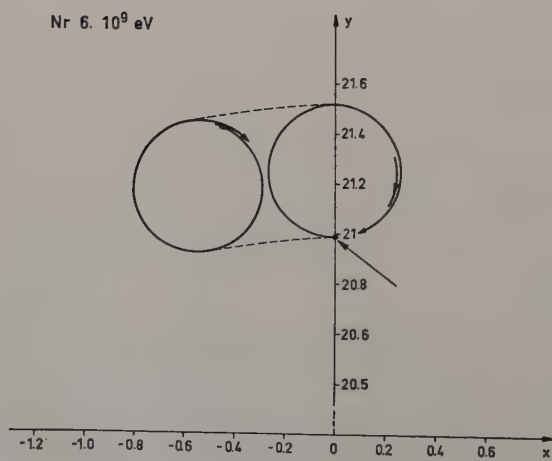






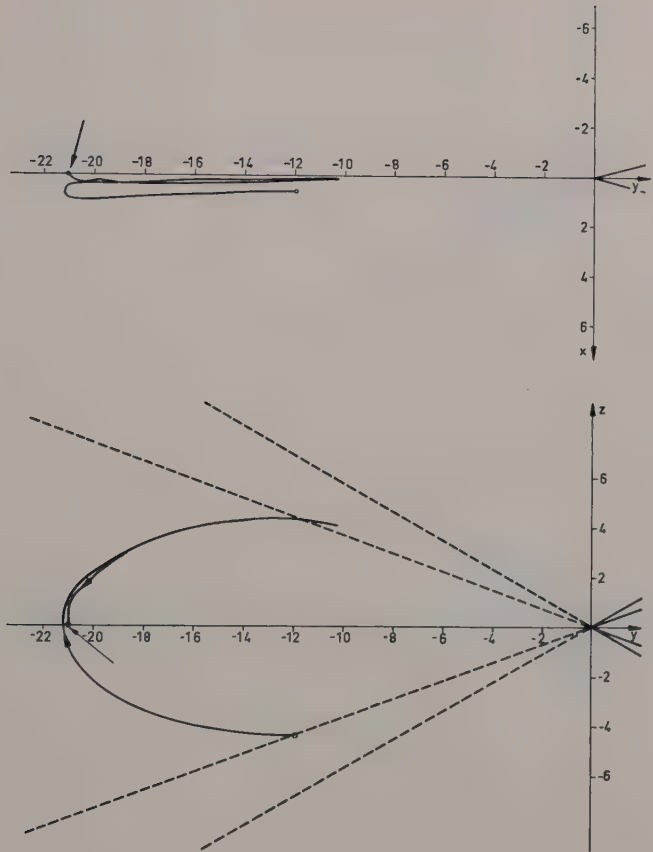


Pl. IV.

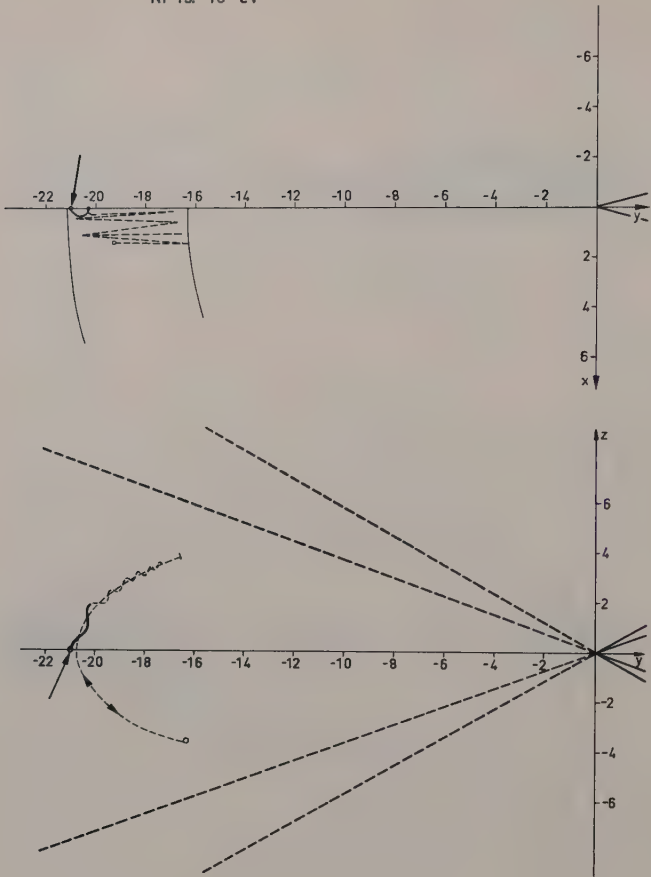


Pl. V.

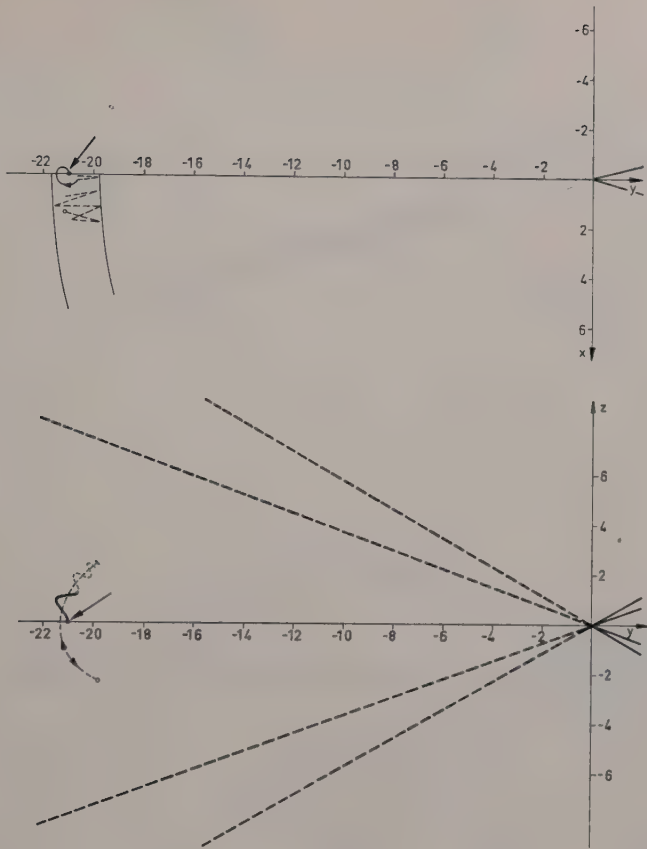
Nr 14. 10^9 eV



Nr 15. 10^9 eV

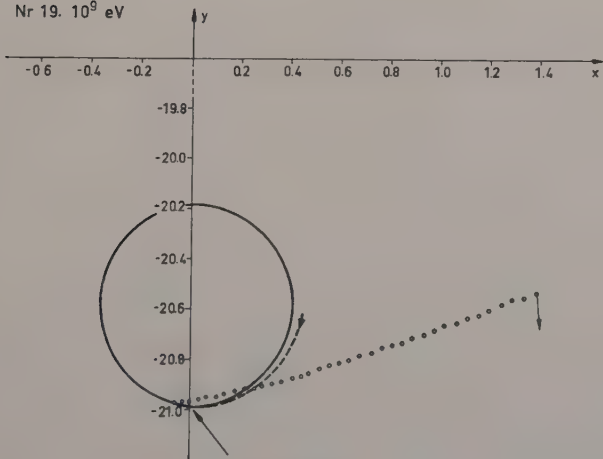


Nr 17. 10^9 eV

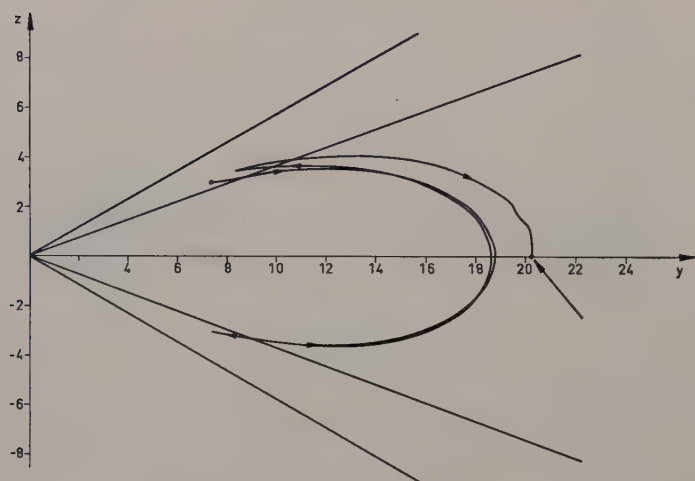
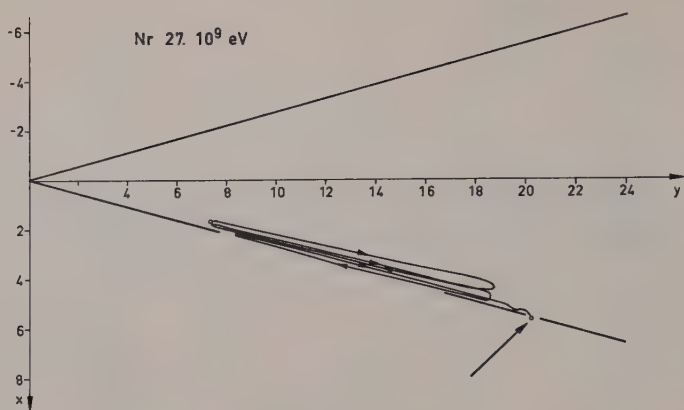


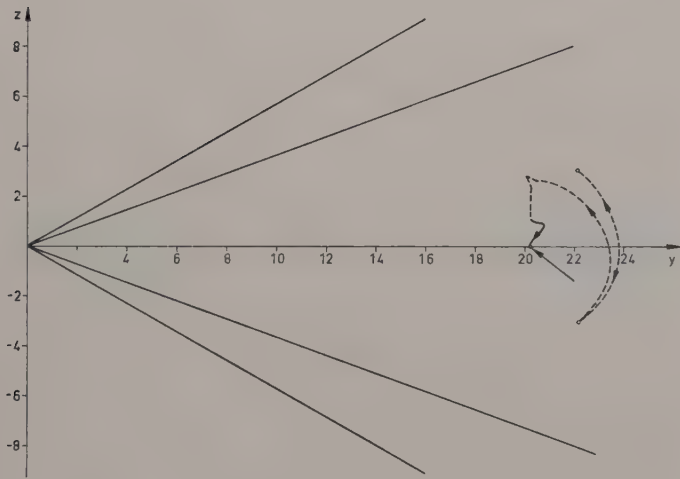
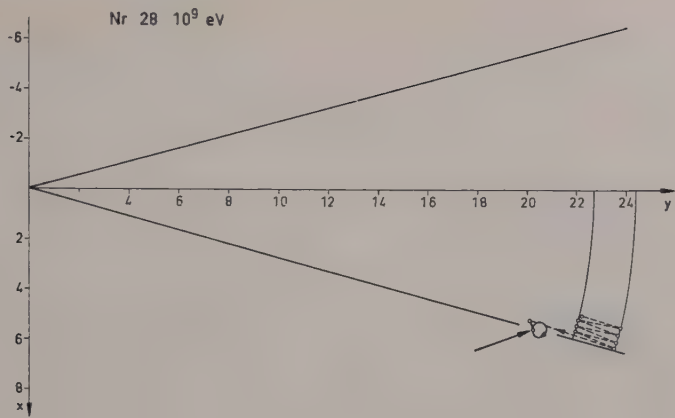
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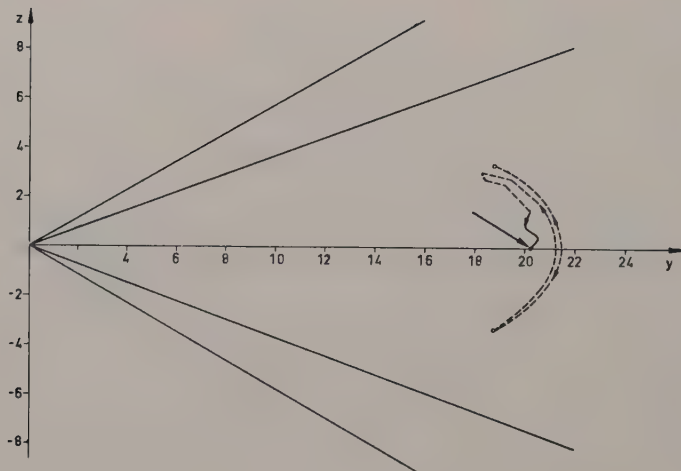
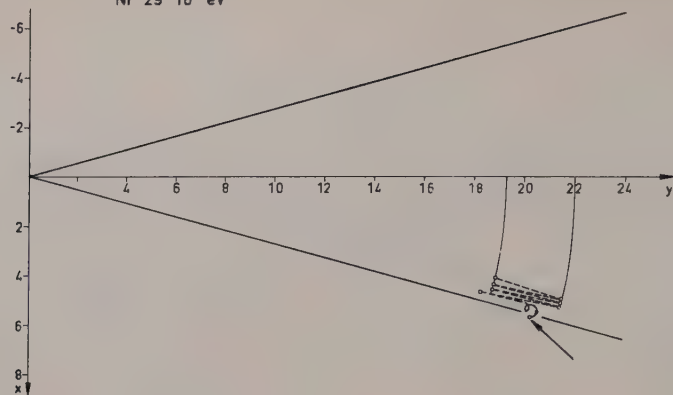
Nr 19. 10^9 eV

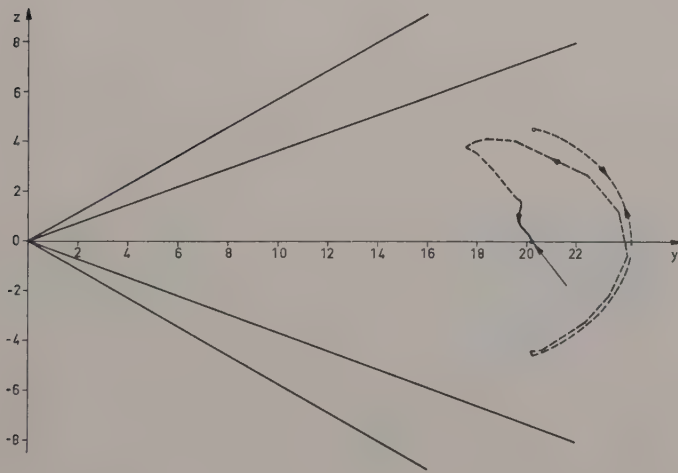
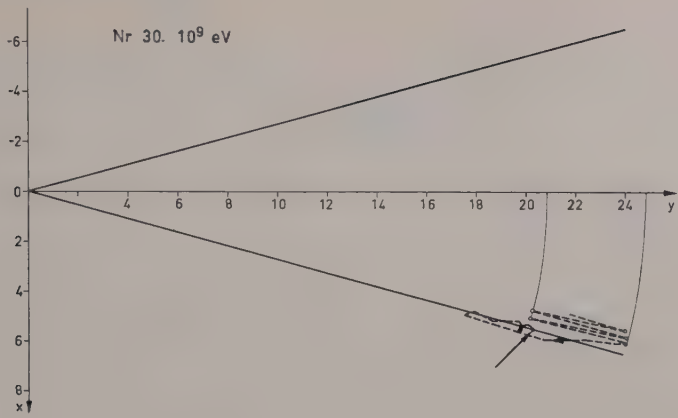


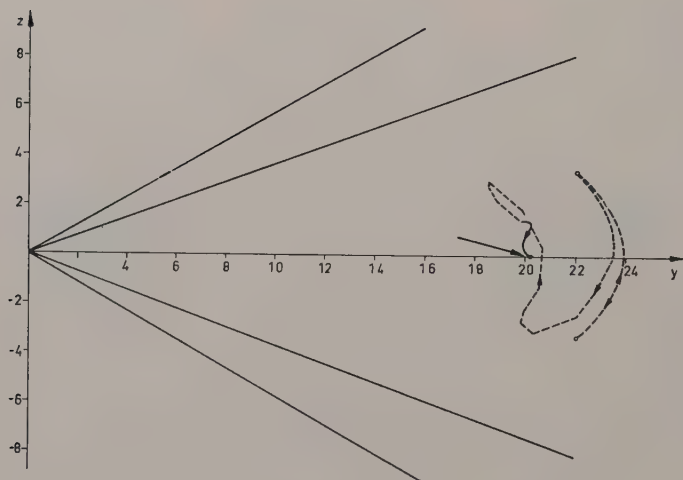
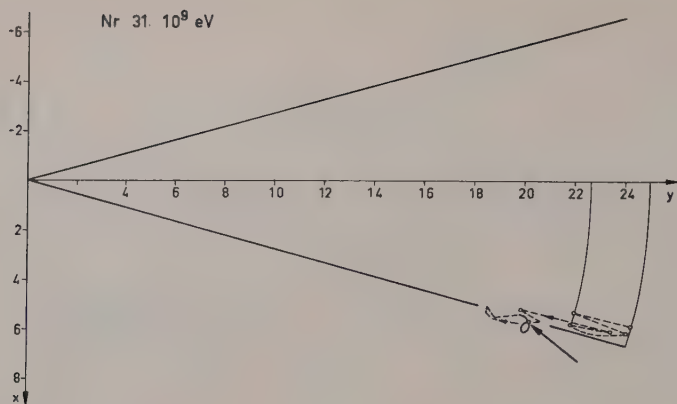
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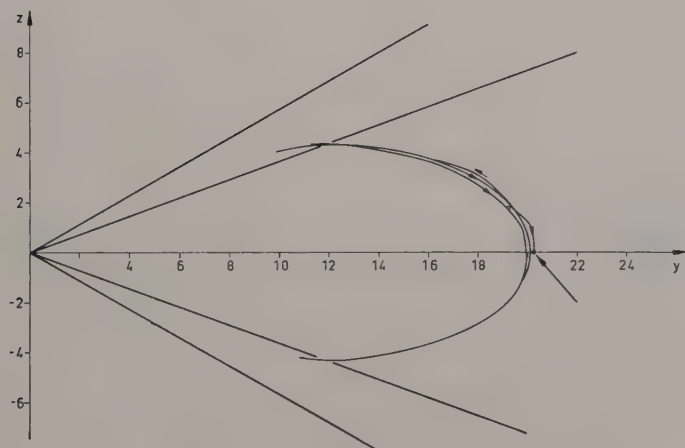
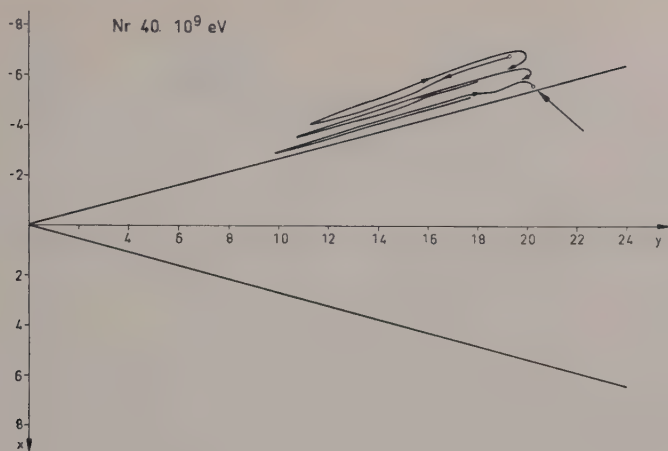


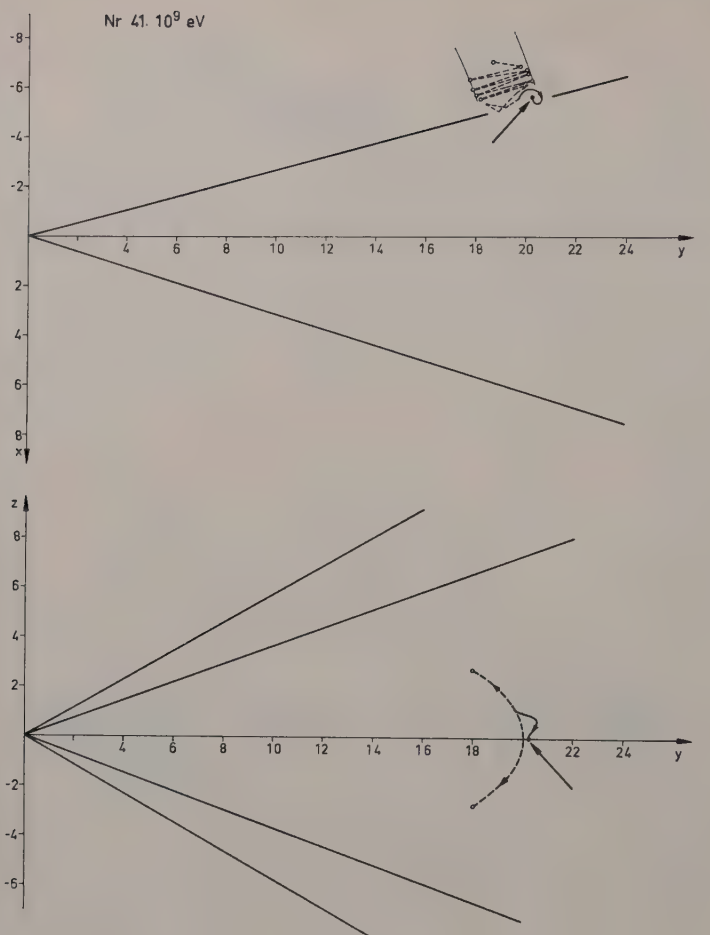


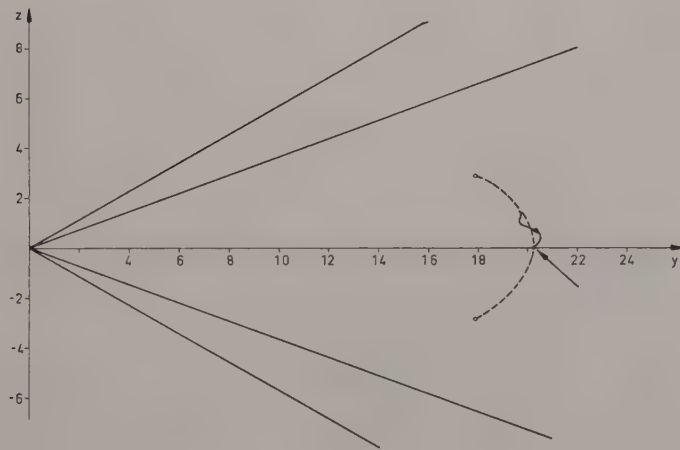
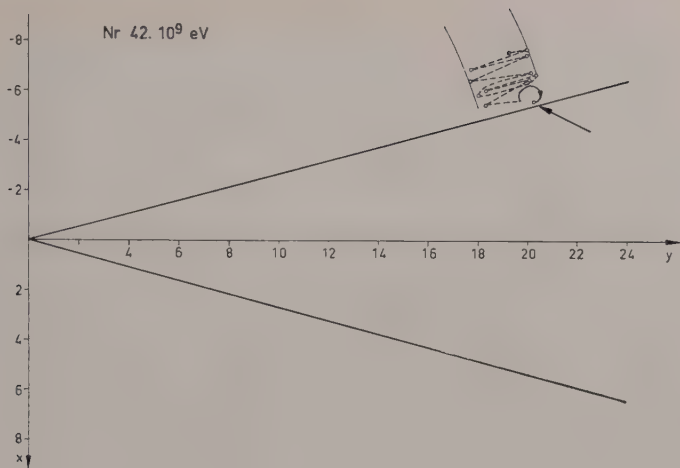
Nr 29 10^9 eV

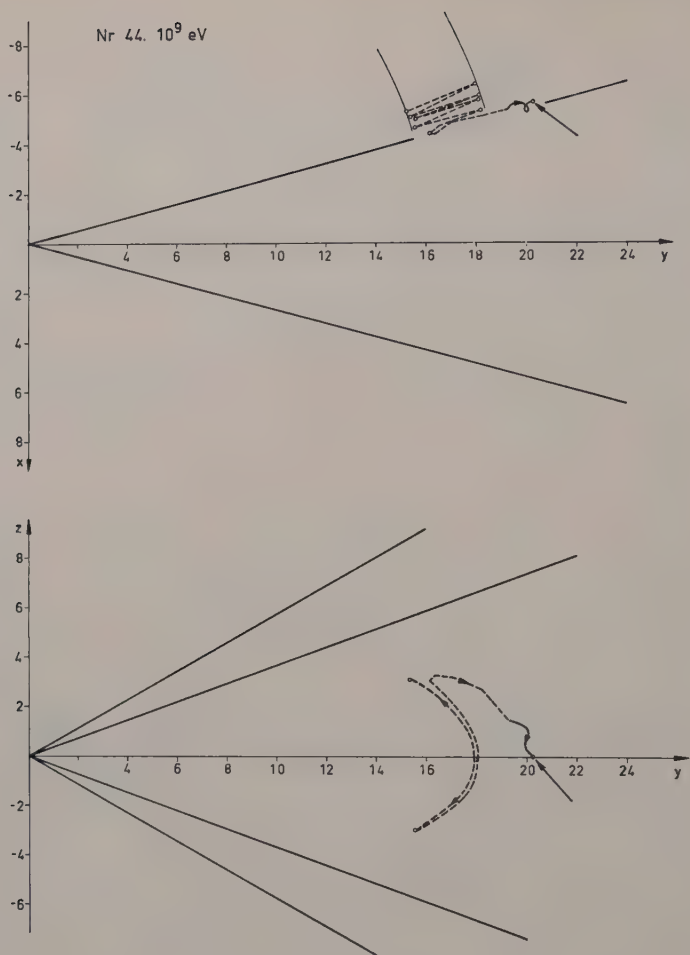


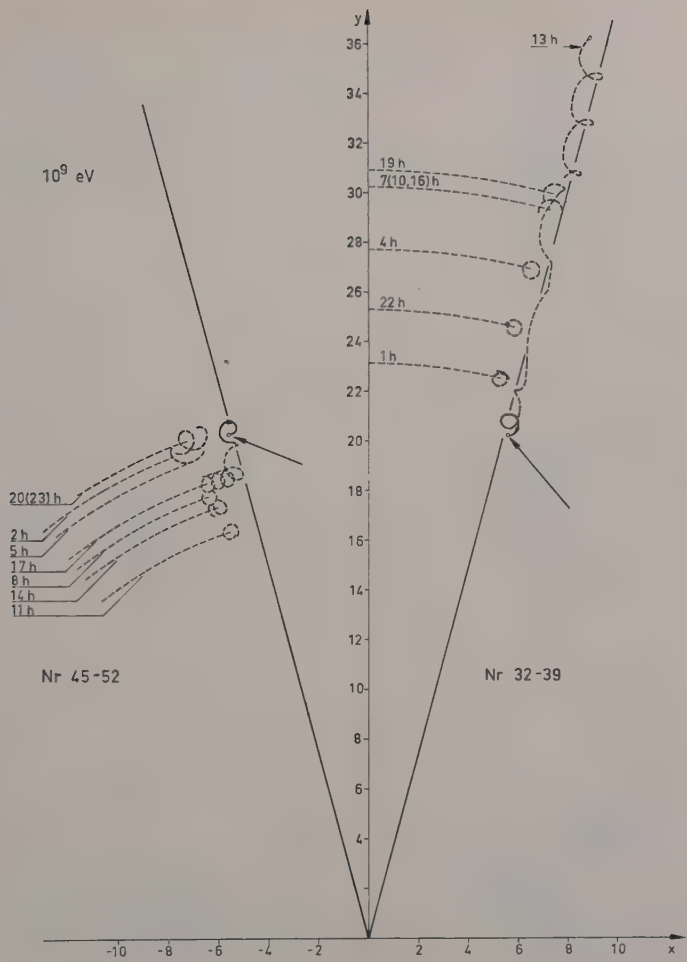


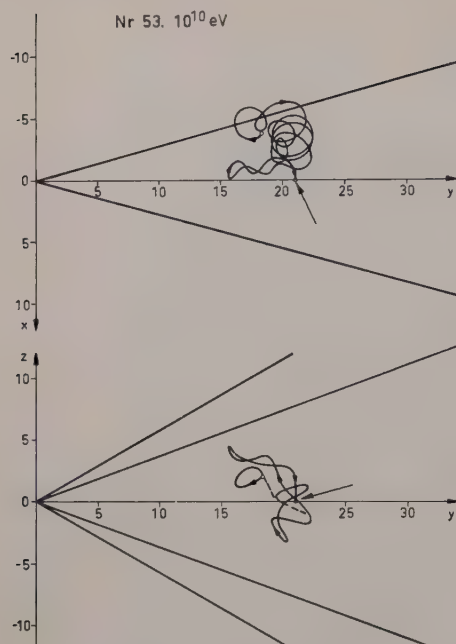




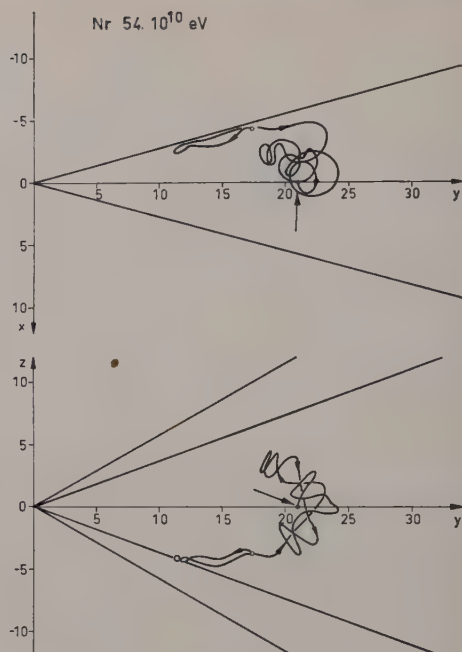




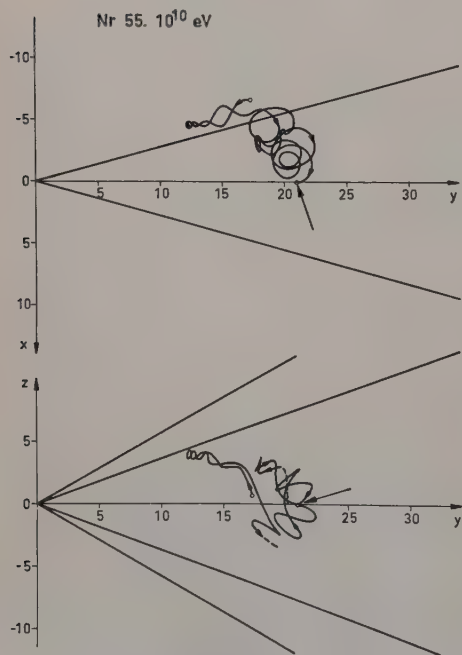




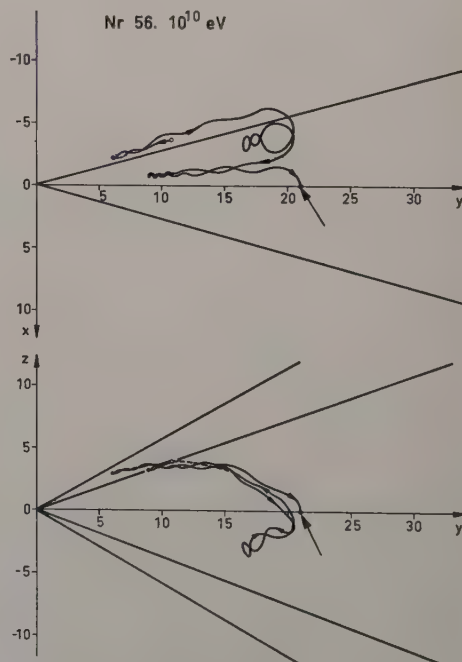
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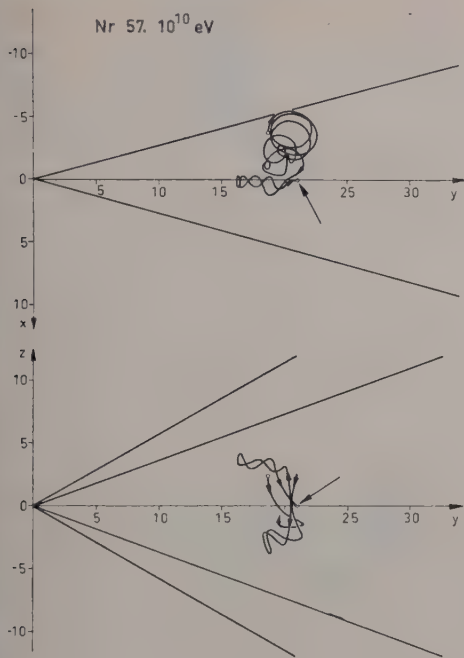
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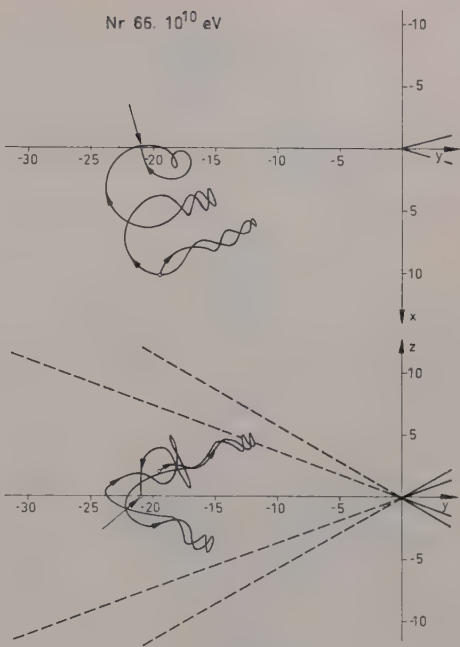
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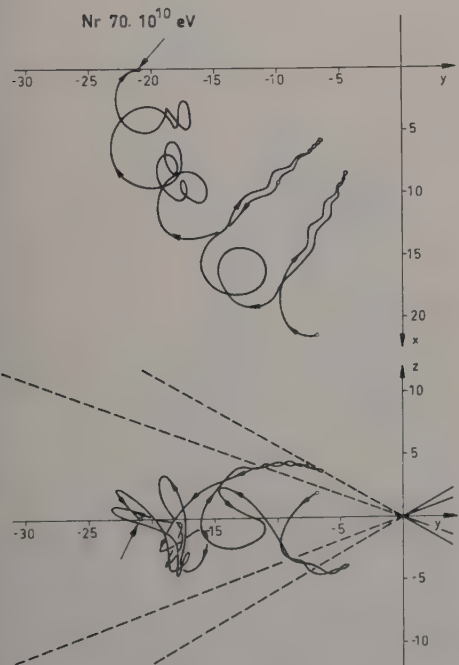
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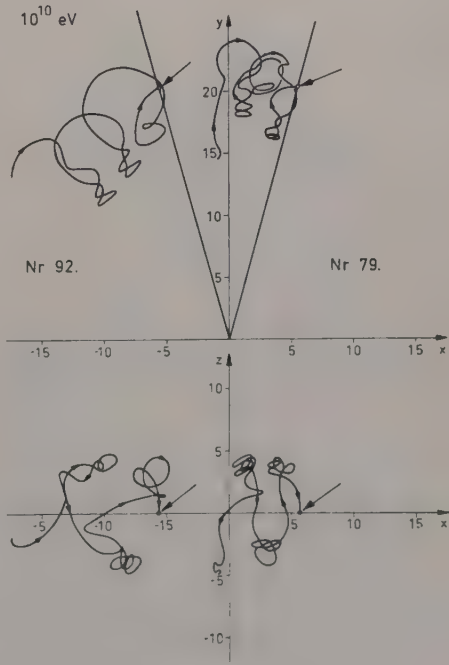
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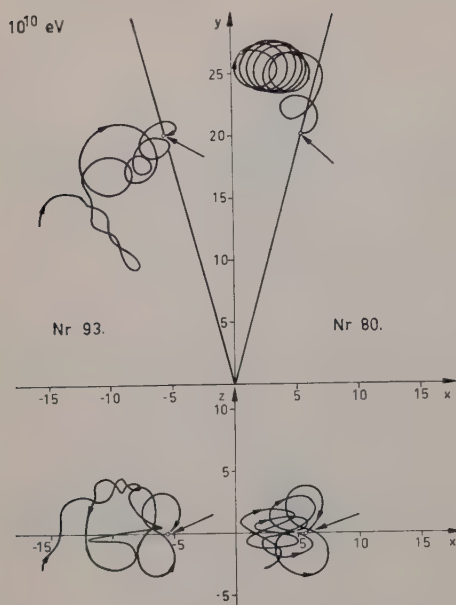
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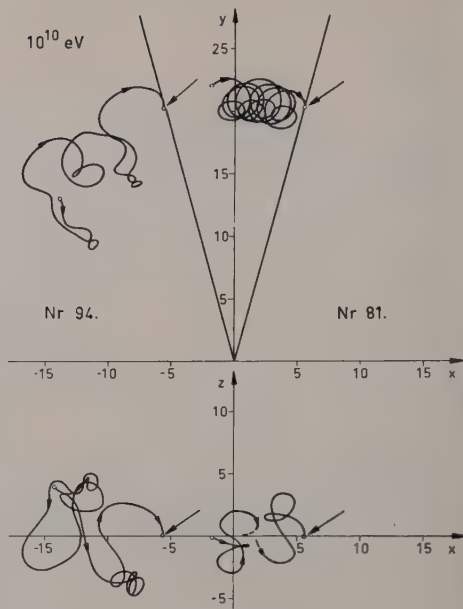
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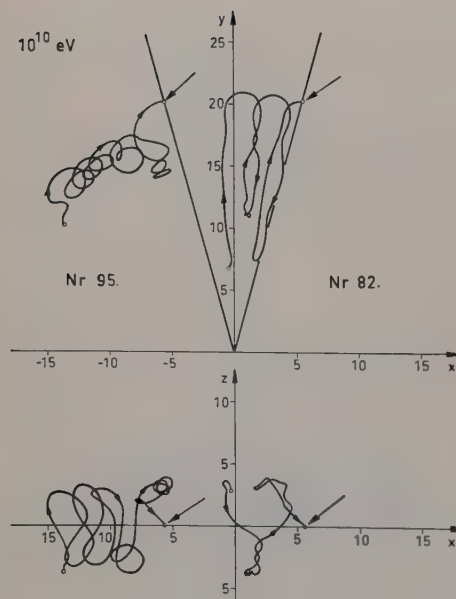
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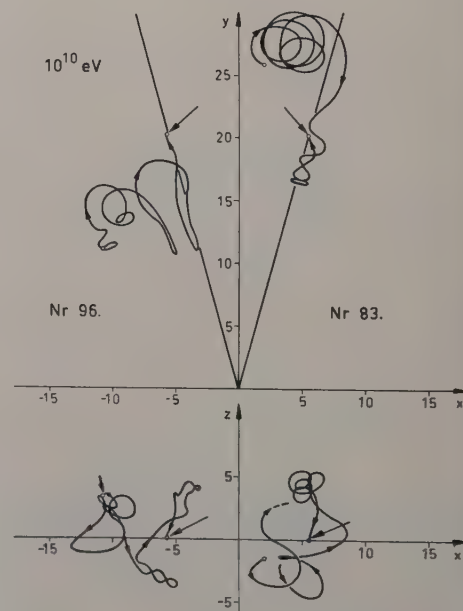
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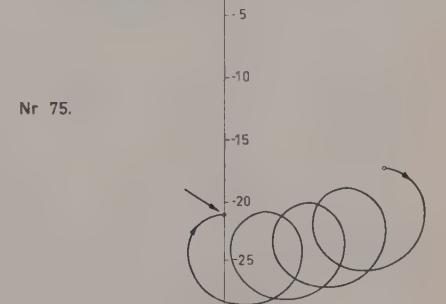
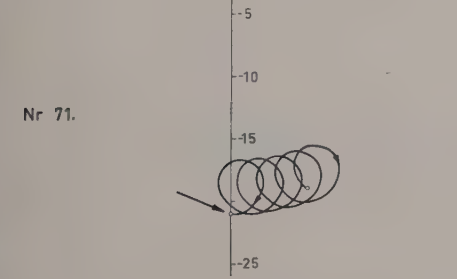
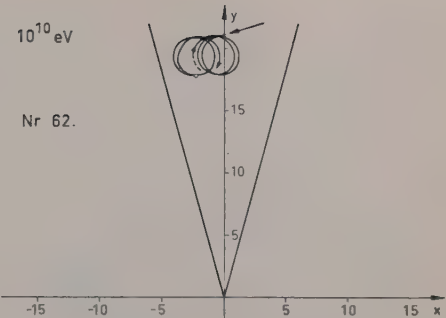
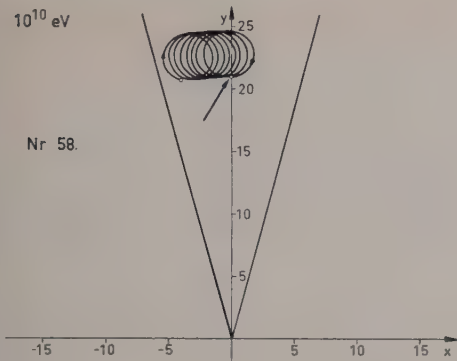
Pl. XXIX.



Pl. XXX.

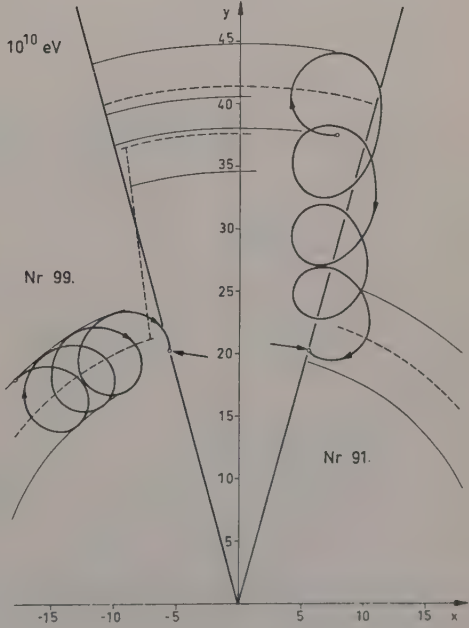
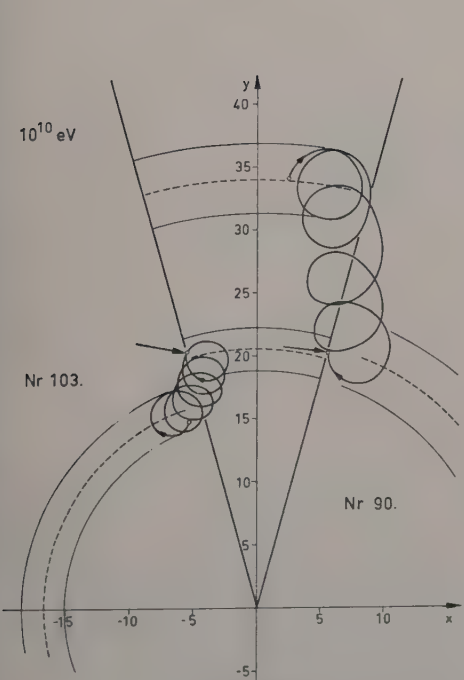


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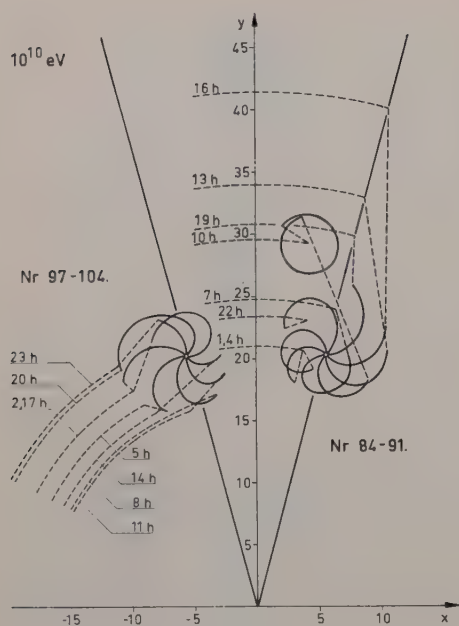
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Pl. XXXIII.

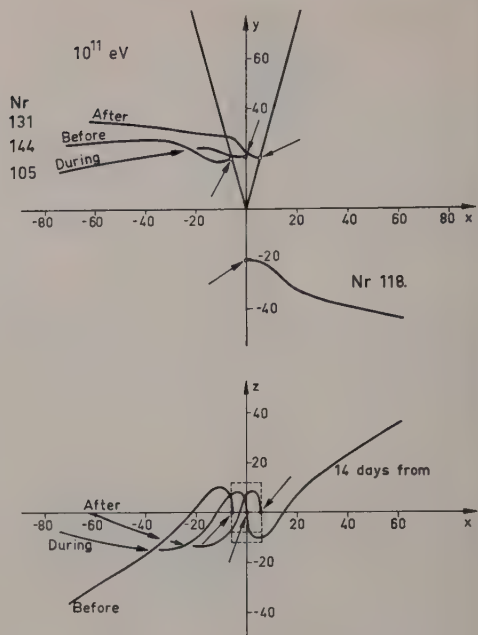


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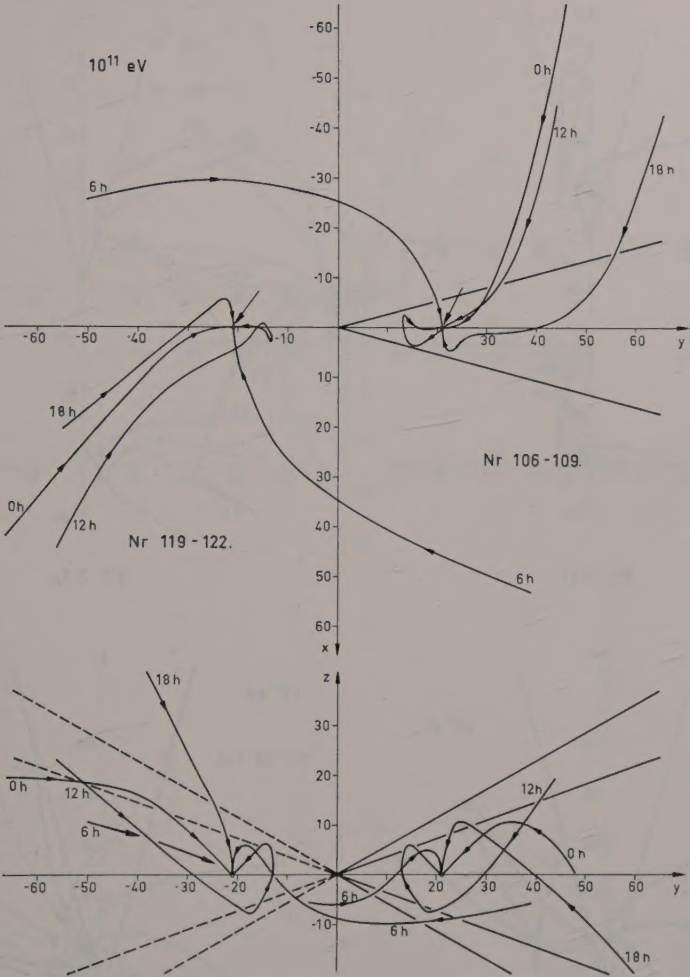
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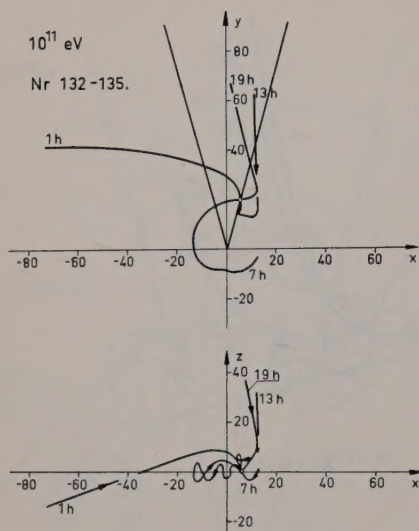


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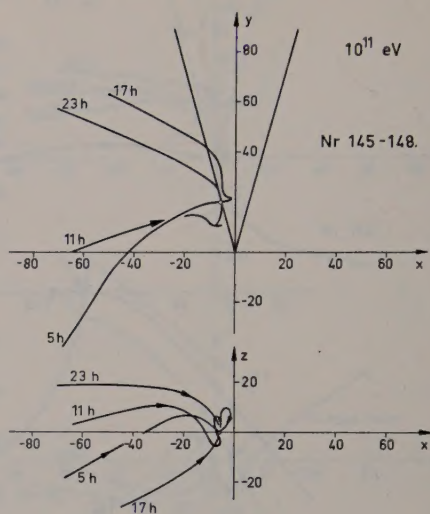


Pl. XXXVII.

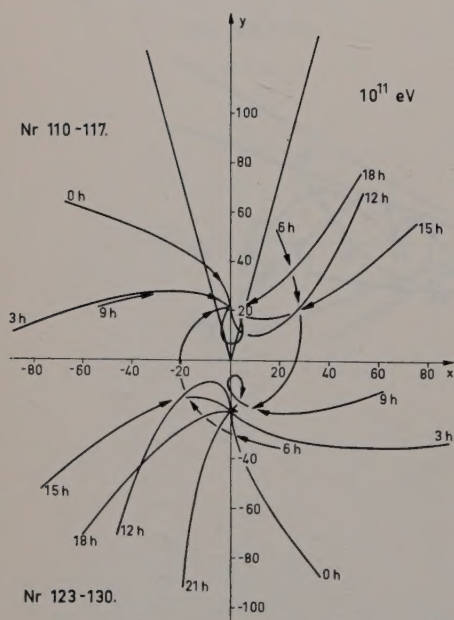




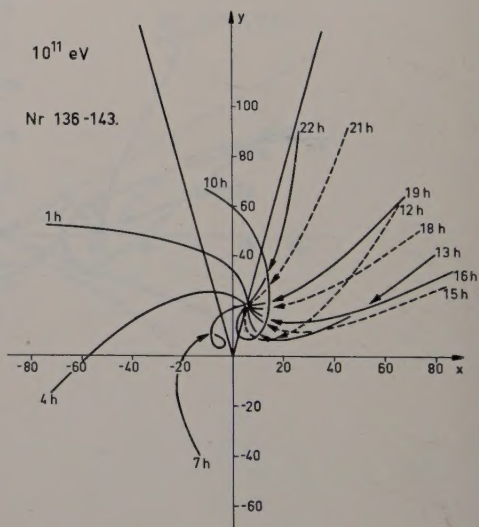
Pl. XII.



Pl. XL.



Pl. XLI.



Pl. XLII.

